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The optimal trade-off between quality and quantity
with uncertain child survival

Thomas Baudin

A blue curved line graphic that starts above the 'C' and ends below the 'E' of the word 'CORE'.

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DISCUSSION PAPER

Center for Operations Research
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Voie du Roman Pays, 34
B-1348 Louvain-la-Neuve
Belgium

<http://www.uclouvain.be/core>

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**The optimal trade-off between quality and quantity
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Thomas BAUDIN¹

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Abstract

The present paper investigates a standard model of endogenous fertility when child survival to adulthood is uncertain. In this framework, I first show that facing the risk their children die before reaching adulthood, parents don't always formulate a precautionary demand for children. Indeed, there exists a non-empty set of utility functions for which parents undershoot their number of children rather than overshooting it. Second, the properties of the optimal economic policy will crucially depend on the manner the Social Welfare Function takes uncertainty into account. More precisely, if Social Welfare is evaluated after the resolution of uncertainty, the parental response to uncertainty is a source of social inefficiency. Then, individual decisions have to be corrected through tax or transfer on both births and education. This property becomes crucial to determine the optimal public response to a mortality crisis in presence of positive externalities on education.

Keywords: fertility, uncertain child survival, optimality conditions, family policy.

JEL Classification: D10, H21, J13, J18

¹ Université catholique de Louvain, CORE and IRES, B-1348 Louvain-la-Neuve, Belgium.
E-mail: Thomas.baudin@uclouvain.be

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1 Introduction

Since the funding contributions of Eckstein and Wolpin [1985] and Nerlove *et al* [1986], a large economic literature explores the optimality of fertility behaviors. The present paper studies the optimality conditions in a standard model of endogenous fertility when child survival to adulthood is uncertain. In this framework, I show two important results. First, facing the risk their children die before reaching adulthood, parents don't always formulate a precautionary demand for children. Second, the properties of the optimal economic policy will crucially depend on the manner the Social Welfare Function takes uncertainty into account. More precisely, if the Social Planner isn't purely utilitarian, the parental response to uncertainty about child survival will have to be corrected by distortive taxes.

Assuming that child survival to adulthood is uncertain makes sense only if the young adult mortality is significantly different from zero.¹ This is the case for a large set of countries. For instance, Mesle and Shkolnikov [1996] show that the Russian mortality crisis has dramatically increased the death rate among young adults. Baland and Estevan [2007] find that young adult mortality is significantly positive for a larger range of countries including for instance Côte d'Ivoire, Zimbabwe and Guatemala. A recent literature points out that the AIDS epidemic has either increased or maintain young adult mortality rate at a high level.² It implies that mortality among young adult is an important issue for individual fertility decisions and therefore, for the nature of the optimal family policy in the economy.³

In the present paper, I use a "standard model of fertility". Since the fundamental contributions of Becker [1973], Becker and Tomes [1976], Becker and Lewis [1973], Becker and Barro [1988], Razin and Ben Zion [1975] and Galor and Weil [1999], the representation of fertility behaviors admits a unified framework. This framework consists of two fundamental assumptions. First, parents value the number of their children (their quantity) as well as

1. For the most developed countries, the young adult mortality isn't very different from zero (see for instance Murray and Lopez [1997]). In this case, adopting a model without uncertainty does not consist in a too strong approximation of reality especially emphasizing that, at the individual level, fertility is a non-negative integer.

2. See for instance Boucekkine *et al* [2009], Buthelezi *et al* [2008], Feeney [2001] and Boerma *et al* [1998]. It is important to note that in the present paper, I don't take into account the problem of orphans that is directly connected to young adult mortality. I assume that young adults die before having any child.

3. The importance of young adult mortality for the explanation of fertility behaviors has been stressed, for instance, by Erlich and Lui [1991], Cervellatti and Sunde [2007], Lagerlof [2003], Kalemli-Ozcan [2003] and Belle and Gersbach [2009]. I only focus here on young adult mortality. Some papers specifically study other dimensions of mortality. See for instance, Boucekkine *et al.* [2003,2009], Zhang and Zhang [2006] and Cervellatti and Sunde [2009] who explore the impact of adult mortality or longevity on fertility decisions. Another set of papers study the impact of child mortality on parental fertility. See for instance, Barro and Becker [1988], Doepke [2005], Kalemli-Ozcan [2008], Strulik [2004] and Soares [2005]. Some papers like Chakraborty [2004] make endogenous the evolution of mortality.

their quality because they are altruistic toward their offsprings. Second, whatever the form of altruism that is assumed, parents maximize their expected utility subject to a non linear budget constraint. Because quality is provided to each child, its cost crucially depends on the quantity choices. Then the parental budget constraint is no longer linear and, a trade-off between quality and quantity takes place.

I adopt here an assumption of perfect altruism as in Razin and Ben Zion [1975] such that parental preferences are dynastic. In this framework, a particular form of additively separable utility is assumed: the utility flow coming from the quantity of children is separable from the utility flow coming from altruism (quality of children) while it is not always separable from the utility flow coming from consumption.⁴ Adopting this form of dynastic preferences enables me to obtain tractable results.

In this framework, I assume that children die before reaching adulthood but after receiving education⁵ and I show two important results. First, facing uncertainty about the number of their children who will survive to adulthood, parents do not always have a precautionary demand for children.⁶ Indeed, parents engage in insurance strategy (hoarding) only when their utility loss coming from their risk aversion is reduced by an additional birth. I show that a non-empty set of utility functions does not satisfy this condition and so, parents undershoot their number of children (they have less children than optimal in a certain environment). For instance, at the steady state, parents have a precautionary demand for children when their utility function is log-linear. However, they undershoot their number of children when their utility function has a weak and constant elasticity of substitution between consumption and the quantity of children.

This result consists in a non trivial generalization of Kalemli-Ozcan [2003] who also

4. A standard alternative consists in assuming a perfect altruism as in Barro and Becker [1988] where the utility flow coming from parental consumption is separable from the utility flow coming from both quantity and altruism which are not separable from each other. In fact, Nerlove and Rault [1997] show that these two models are specific cases of a more general model where the utility flows coming from consumption, fertility and altruism are not separable. This general model is briefly presented in section 2. In the context of uncertain child survival, the Razin and Ben Zion's model enables to use both standard approximation methods of utility under uncertainty and the concepts of prudence a la Kimball [1990] and Leland [1968] since fertility directly enters the instantaneous utility function. This is not the case for the Barro and Becker's model (see section 2). Nevertheless, it is important to keep in mind that the present model remains an ad-hoc specification of the most general endogenous fertility model what can drive some of my results.

5. A large set of papers also adopt this assumption, see, for instance, Cervellatti and Sunde [2007], Kalemli-Ozcan [2003], Lagerlof [2003], Erlich and Lui [1991], Bell and Gersbach [2009]

6. In line with Kalemli-Ozcan [2003], parents have a precautionary demand for children if, facing the risk that children die, they have more children than optimal in a certain environment. In other words, they overshoot their optimal number of children in order to ensure that they won't have less surviving children than optimal.

assumes that children die after receiving education from their parent.⁷ In her framework, parents are characterized by log-linear preferences. Then, when the number of surviving children is uncertain, parents always have a precautionary demand for children and never undershoot their fertility rate. I show that this is not verified for any utility functions.⁸

The second main result of this paper comes from the determination of the optimal family policy when child survival to adulthood is uncertain.⁹ Intuitively, because there is no externality, if the social planner is purely utilitarian the competitive equilibrium coincides with the first best path. However, as shown by Gajdos and Maurin [2004], in presence of uncertainty, the problem of the timing effect arises: should Social Welfare be evaluated before or after the resolution of uncertainty? Assuming a purely utilitarian social planner requires to evaluate the social welfare before the resolution of uncertainty. The SWF is then *ex-ante*, it equals the utility of the representative agent before the resolution of uncertainty. Despite this assumption is consistent with the aggregation of utilities, another way to define the SWF under uncertainty consists of the *ex-post* SWF. In this case, the social welfare is evaluated after the resolution of uncertainty. Then the SWF equals the utility of the representative agent after the resolution of uncertainty.¹⁰

Here, because the law of large number applies, the average number of surviving children

7. Assuming young adult mortality implies that parents cannot implement a replacement strategy. In a more recent paper, Kalemli-Ozcan [2008] assumes that children die before receiving education what is compatible with the assumption of child mortality.

8. Kalemli-Ozcan [2003,2008] propose a model of long-run growth what requires to obtain closed-form solutions. It prevents from obtaining more general conclusions on individual behaviors. It is also important to note that she adopts an assumption of imperfect altruism (warm-glove motive) leading to non-dynastic preferences. However, I show in section 2 that her results are valid in the dynastic framework of Razin and Ben-Zion.

9. Eckstein and Wolpin [1985] and Nerlove *et al* [1986] are the first paper to study the optimality conditions in a model of trade-off between quality and quantity. Obviously, they don't emphasize uncertain child survival. In this paper, I will deal only with Millian type SWF, that is to say with maximizing the welfare of the representative parent. See Nerlove *et al* [1986] and Baudin [2010] for an analysis of the Benthamite case, and Spiegel [1993] who deal with Rawlsian SWF.

10. Gajdos and Maurin provide an interesting example to show why using both *ex-ante* and *ex-post* SWF is very important to analyze the optimal equilibrium in an economy subject to uncertainty. They also quote an enlightening sentence of Myerson [1981]:

"The moral of this story is that simply specifying a social welfare function may not be enough to fully determine a procedure for collective decision making. One must also specify when the individuals' preferences or utility levels should be evaluated; before or after the resolution of uncertainties. The timing of social welfare analysis may make a difference. The timing-effect is often an issue in moral debate, as when people argue about whether a social system should be judged with respect to its actual income distribution or with respect to its distribution of economic opportunities" (p. 884).

See also Sandmo [1983].

is known with almost certainty at each period.¹¹ This implies that the *ex-post* SWF consists of the utility derived from the average number of surviving children at each date. Obviously, adopting this representation of social preferences leads to different results compared with the *ex-ante* SWF. Indeed, in this case, the Social Planner can be viewed as risk neutral since he maximizes the utility of the representative parent having the expected number of children rather than the parental expected utility. It implies that the parental insurance strategy against risk becomes a source of social inefficiency. Therefore, if parents overshoot their optimal fertility rate, they have too much children and it is optimal to tax births. Because the parental budget constraint isn't linear, the tax on births increases the marginal cost of human capital for future generations and so, it is optimal to subsidize education. Conversely, if parents engage in undershooting, they don't have enough offspring at the competitive equilibrium. Then, it is optimal to subsidize births and because the budget constraint isn't linear, it is also optimal to tax investments in education.

This result will become crucial once externalities are introduced in the model. Indeed, a large set of papers assume that there exist positive externalities in the production of human capital.¹² Baudin [2010] shows that in presence of positive externalities in the production of human capital, the optimal economic policy in a Millian economy consists of subsidizing education and taxing births. This result crucially comes from both the Lucas-type externality and the non-linearity of the parental budget constraint. Because of the Lucas-type externality, parents don't take into account all the returns of their investment in their children's human capital. Therefore, they tend to underinvest in the quality of children such that education spending has to be subsidized. However, the subsidy on education reduces the marginal cost of the quantity of children. Then parents have too much children and births have to be taxed.

Intuitively, the existence of uncertainty about child survival to adulthood could alter the nature of the optimal economic policies as well as the optimal response to a mortality crisis. I propose an extension of the Benchmark model by introducing a Lucas type externality in the accumulation of human capital. In this new model, if the Social Planner is purely utilitarian (*ex-ante* SWF), the Lucas-type externality implies that births have to be taxed while education have to be subsidized as in Baudin [2010].¹³

When the SWF is *ex-post*, two effects interact: (*i*) the Lucas-type externality necessitates

11. The population is assumed to be large enough to make this approximation a precise one.

12. See, for instance, Galor and his co-authors [1999, 2005], De la Croix and Doepke [2003] and Lucas [1988].

13. With a purely utilitarian Social Planner, there is no difference between individual and social preferences. Then, the only motive for implementing an economic policy is the Lucas type externality.

to tax births and subsidize education and (ii) the parental strategy against risk is a source of inefficiencies. If parents have a precautionary demand for children, they have too much children what requires to tax births and to subsidize education. Then, overshooting reinforces the impact of the Lucas-type externality. However, if parents undershoot their fertility rate, they tend to have too little children such that births should be taxed. Then, the two main effects go in the opposite direction. As a result, if the Lucas type externality is strong, the undershooting effect is fully dominated by the externality effect and it is optimal to subsidize education and to tax births. Conversely, if the Lucas type externality is weak, the undershooting effect fully dominates the externality effect and it is optimal to tax education and to subsidize births. For intermediary intensities of the educational externality, it becomes optimal to subsidize both births and education.

The rest of the paper is organized as follows. Section 2 describes the laissez-faire equilibrium and determines the conditions under which parents have a precautionary demand for children. Section 3 describes the social optimum while Section 4 determines the optimal economic policy. Then, I extend the model to positive externalities of human capital in Section 5. Section 6 concludes.

2 Competitive Equilibrium and Precautionary Demand For Children

In this section, I first present the model economy and its competitive equilibrium. Then, I show that, facing uncertainty about their children survival to adulthood, parents don't always have a precautionary demand for children.

2.1 The Laissez-Faire Equilibrium

The model consists of an overlapping generations economy with L_t adult agents. Each agent can live potentially two periods: childhood and adulthood. As in Lagerlof [2003], Baland and Esteban [2006] and Erlich and Lui [1991], each child has a probability q_t to reach adulthood. This probability is assumed to be exogenous. Children do not consume commodities but receive education from their parent. This educational investment is denoted e_t , it increases the children's future human capital h_{t+1} if he or she becomes adult. The accumulation of

human capital follows a standard process:

$$h_{t+1} = f(e_t, h_t) \quad (1)$$

Function f is strictly increasing and concave with respect to all its arguments. I assume non increasing returns to scale. An intrafamily transmission of human capital takes place: the parental human capital h_t positively influences the future human capital of children. This is a quality effect of the schooling time.

When a child born in $t - 1$ reaches adulthood, he has to decide his consumption level c_t , the number of his children N_t and their education e_t . Notice that parents decide their number of births N_t but not their number of surviving children $n_t \leq N_t$ that is a random variable. In line with Sah [1991] and Kalemli-Ozcan [2003], n_t follows a binomial distribution. The probability that n_t children out of N_t births survive, is:

$$j(n_t, N_t, q_t) = \binom{N_t}{n_t} q_t^{n_t} (1 - q_t)^{N_t - n_t} \quad (2)$$

where q_t is the probability for each child to survive to adulthood. Because the law of large numbers applies and agents are identical, the population's law of motion is simply:

$$L_{t+1} = q_t N_t L_t \quad (3)$$

For simplicity's sake, families are mono-parental. The familial budget constraint is:

$$c_t = [1 - (\phi + \theta e_t) N_t] w_t h_t \quad (4)$$

where $(\phi, \theta) \in]0, 1[^2$ denote respectively the fraction of the parent's time endowment (normalized to one) that is needed to rear one child and the time cost of one unit of education. The total opportunity cost to have N_t children is then equal to $(\phi + \theta e_t) w_t h_t N_t$ where w_t is the wage rate per effective worker. The cost of one unit of education isn't affected by the child mortality rate since a child death is assumed to occur after the education process.¹⁴

14. In a more general framework, I should also include child mortality in addition to young adult mortality. To do so, I should introduce the probability for a child to die before age five. In this case, the child mortality would affect the cost of education since education is provided after age five. Doing so would not alter my main result if I assumed, as in Baudin [2010], that child mortality isn't a source of uncertainty what is a reasonable assumption.

The final good is produced in quantity Y_t following a linear technology:

$$Y_t = AH_t \quad (5)$$

A is a productivity factor and H_t is the total amount of human capital in the workforce. At the labor market's equilibrium, H_t is:

$$H_t = [1 - (\phi + \theta e_t) N_t] h_t L_t \quad (6)$$

The workforce participation of a parent consists of his remaining time after childbearing and educating his or her children. Furthermore, as the labor market is competitive, the wage equals the workers' marginal productivity at each date t :

$$w_t = A \quad (7)$$

As in Razin and Ben Zion [1975], parents are altruistic toward their children such that their preferences are dynastic. Let V_t denote the maximal expected utility of an adult born in $t - 1$ such that:¹⁵

$$V_t = \max \left\{ \sum_{n_t=0}^{N_t} j(n_t, N_t, q_t) u(c_t, n_t) + \beta V_{t+1} \right\} \quad (8)$$

where $u(c_t, n_t)$ is the current utility of the representative parent having n_t surviving children. β denotes the parental discount rate. $u(.,.)$ is strictly increasing and concave in its arguments. In this representation à la Razin and Ben Zion, I assume that parent's utility from consumption is separable from her child's lifetime utility but not from her fertility choice.¹⁶ Parents value the number of surviving children, this means that child mortality is a source of disutility. Because child survival is uncertain, parents have to maximize their expected utility.

I assume that parents are characterized by rational expectations. $E_{t-i}(g_t)$ denotes the

15. In the model of Razin and Ben Zion, there is no uncertainty about the number of surviving children. Then, the parental utility function is $\Psi_t = \max \{u(c_t, n_t) + \beta \Psi_{t+1}\}$. In the present paper, V_t consists of an adaptation of this utility function to uncertain child survival.

16. Alternatively, I could assume, as in Barro and Becker [1988], that fertility choice is not separable from the child's lifetime utility while it is from the parent's consumption. This alternative has been widely used in the literature. Nerlove and Rault [1997] show that, in fact, the Barro and Becker's model, as well as the model designed by Razin and Ben Zion, are specific cases of a more general model. They present this more general model where, sidestepping uncertainty, $V_t = \max \{u(c_t, n_t) + \beta(n_t) n_t V_{t+1}\}$. In the Becker and Barro specification, $u'_{n_t} = 0$ while in the Razin and Ben Zion model, $n_t \beta(n_t) = \beta$. Jones and Schoonbroodt [2009] and Bar and Leukhina [2010] show that for some joint restrictions on $\beta(n_t)$ and $u(c_t, n_t)$, both models are identical.

expectation of g_t at date $(t - i)$. The assumption of rational expectations implies that $E_{t-i}(n_t) = q_t N_t$ and $E_{t-i}(n_t - q_t N_t)^2 = q_t(1 - q_t)N_t$. Furthermore, parental expectations are assumed to be perfectly observable by the Social Planner and the government. Iterating V_t to infinity, I obtain the equivalence with the following centralized problem:¹⁷

$$\max V_t = \sum_{s=t}^{+\infty} \beta^{s-t} \sum_{n_s=0}^{N_s} j(n_s, N_s, q_s) u(c_s, n_s) \quad (9)$$

As in Kalemli-Ozcan [2003], I approximate the parental current utility around the mean of the binomial distribution thanks to Taylor series.¹⁸ A third degree approximation around the mean $E_{t-i} n_t = q_t N_t \forall (t - i, t)$ provides the following result:

$$u(c_t, n_t) \simeq \sum_{\nu=0}^3 u_{n_t}^{(\nu)}(c_t, q_t N_t) (n_t - q_t N_t)^\nu \quad (10)$$

where $u_{n_t}^{(\nu)}(c_t, q_t N_t)$ denotes the ν^{th} derivative of $u(c_t, n_t)$ with respect to n_t evaluated at the point $q_t N_t$ and $u_{n_t}^{(0)}(c_t, q_t N_t) = u(c_t, q_t N_t)$. For simplicity's sake, I assume that fourth and higher order terms are negligible what is a reasonable assumption. Because of the parental rational expectations, $E_{t-i}(n_t - q_t N_t)^\nu = 0 \forall \nu \in \{2n + 1; n \in \mathbb{N}\}$ and $E_t(n_{t+i} - q_{t+i} N_{t+i})^2 = q_{t+i}(1 - q_{t+i})N_{t+i}$ is the variance of the binomial.¹⁹ Therefore, the maximal expected utility of a parent born in $t - 1$ can be written as follows:²⁰

$$\max V_t \simeq \sum_{s=t}^{+\infty} \beta^{s-t} \left[u(c_s, q_s N_s) + \frac{q_s(1 - q_s)N_s}{2} u_{n_s n_s}''(c_s, q_s N_s) \right] \quad (12)$$

17. A necessary condition to obtain such an equivalence is: $\lim_{T \rightarrow +\infty} \beta^T \sum_{n_T=0}^{N_T} j(n_T, N_T, q_T) u(c_T, n_T) = 0$. n_T being the number of surviving children, it is bounded by $\frac{1}{\phi}$ the maximal number of children a wife can give birth to. So, this condition will always be satisfied when income and so consumption are bounded at the steady state what will be the case in the rest of the paper. Indeed, in this case, the expected utility of a parent is also bounded.

18. Notice that Sah [1991] also assumes binomial distributions but doesn't proposes approximations with Taylor series.

19. From now, n is being used to denote the number of surviving children that is no more a natural number.

20. Here again, the objective function is bounded. Indeed, $q_t \in [0, 1]$. Notice that, because I focus on the mortality of young adults, there is no uncertainty about consumption levels. Indeed, the costs of surviving and non surviving children are the same such that, the consumption of parents do not depend on the number of surviving children which is random. If I had assume child mortality, at age five, as a source of uncertainty, the true approximation of the expected parental current utility would have been:

$$u(c_s, n_s) \simeq u(c_s, q_s N_s) + \frac{q_s(1 - q_s)N_s}{2} \left[u_{nn_s}'' + \frac{\partial^2 C_s}{\partial n_s^2} u_{c_s}' + \left(\frac{\partial C_s}{\partial n_s} \right)^2 u_{cc_s}'' + 2 \frac{\partial C_s}{\partial n_s} u_{cn_s}'' \right] \quad (11)$$

As mentioned by Kalemli-Ozcan [2003], this approximation allows to introduce a risk effect (the variance) in a tractable way. Indeed, at each date t , $\frac{q_t(1-q_t)N_t}{2}u''_{n_t n_t}(c_t, q_t N_t)$ is the total utility loss of a risk-averse parent born in $t-1$ due to the uncertainty about child survival.²¹ Let \mathcal{L}_t be the Lagrangian of the representative parent's problem. It follows that

$$\mathcal{L}_t = \sum_{s=t}^{+\infty} \beta^{s-t} \left[u(c_s, q_s N_s) + \frac{q_s(1-q_s)N_s}{2} u''_{n_s n_s}(c_s, q_s N_s) + \eta_s (f(e_s, h_s) - h_{s+1}) \right] \quad (14)$$

where η_s is the Lagrange multiplier attached to the accumulation of human capital at each date s . A parent born in $t-1$ will determine his optimal demands (c_t, N_t, e_t) by maximizing (14) substituting c_t by its expression in (4). Thus, I obtain the three following first-order conditions with respect to N_t, e_t and h_{t+1} :²²

$$-(\phi + \theta e_t)w_t h_t \left(u'_{c_t} + \frac{q_t(1-q_t)N_t}{2} u'''_{n_t n_t} \right) + q_t u'_{n_t} + \frac{q_t(1-q_t)}{2} u''_{n_t} + \frac{q_t^2(1-q_t)N_t}{2} u'''_{n_t n_t} = 0 \quad (15)$$

$$-\theta N_t w_t h_t \left(u'_{c_t} + \frac{q_t(1-q_t)N_t}{2} u'''_{n_t n_t} \right) + \eta_t f'_1(e_t, \cdot) = 0 \quad (16)$$

$$w_{t+1}(1 - [\phi + \theta e_{t+1}]N_{t+1}) \left(u'_{c_{t+1}} + \frac{q_{t+1}(1-q_{t+1})N_{t+1}}{2} u'''_{n_{t+1} n_{t+1}} \right) + \eta_{t+1} f'_2(e_{t+1}, \cdot) = \frac{\eta_t}{\beta} \quad (17)$$

The competitive equilibrium is described by the set $\{c_t^*, N_t^*, e_t^*, h_t^*, H_t^*, Y_t^*, w_t^*\}_{t=0}^{t=+\infty}$ satisfying equations $\{(1), (4) - (7), (15) - (17)\}_{t=0}^{t=+\infty}$.²³

In the following sub-section, I explore the conditions that have to be satisfied to observe a precautionary demand for children.

21. The main reason why I didn't adopt the Barro and Becker [1988]'s representation of preferences is that such an approximation would not be tractable. Indeed, adopting this alternative representation of preferences and assuming $L_0 = 1$, I would obtain:

$$V_0 = \sum_{t=0}^{+\infty} \beta^t u(c_t) \prod_{s=0}^{t-1} \left(\sum_{n_s=0}^{N_s} j(n_s, N_s, q_s) n_s^{1-\kappa} \right) \quad (13)$$

with $\kappa \in [0, 1]$ when $u(c_t) > 0 \forall c_t > 0$ and $\kappa > 1$ when $u(c_t) < 0 \forall c_t > 0$ (see Jones and Schoonbroodt [2007]). Using a third degree approximation around the mean of the binomial wouldn't introduce a risk effect (the variance) in a tractable way.

22. To ensure global concavity of the problem, its Hessian Matrix is assumed to be negative semi-definite.

23. The following transversality condition has also to be satisfied:

$$\lim_{t \rightarrow +\infty} \beta^t \frac{\theta N_t A h_t}{f'_1(e_t, h_t)} \left(u'_{c_t} + \frac{q_t(1-q_t)N_t}{2} u'''_{n_t n_t} \right) h_{t+1} = 0 \quad (18)$$

2.2 Do parents always overshoot their optimal fertility rate?

Definition 1 *Parents overshoot their number of children (i.e.: they have a precautionary demand for children) if facing uncertainty about their reproductive success, they decide to give births to more children than optimal in an environment without uncertainty. Conversely, parents will undershoot their number of children if they decide to give births to less children.*

Proposition 1 *Facing uncertainty about the number of children who will survive to adulthood, parents overshoot their fertility rate if:*

- $\frac{u''_{nn_t}}{N_t} > -q_t u'''_{nnn_t}$ when their preferences are separable between consumption and fertility
- $\frac{u''_{nn}}{N} > (\phi + \theta e) w h u'''_{nnc} - q u'''_{nnn} \forall u(c_t, n_t)$ at the steady state

When this condition isn't satisfied, parents undershoot their number of children.

Proof. See Appendix A. ■

In presence of uncertainty about their reproductive success, parents have a precautionary demand for children (hoarding) only if an additional birth reduces their utility loss coming from their risk aversion. This loss is approximated by $\frac{q_t(1-q_t)N_t}{2} u''_{n_t n_t}$.²⁴

It is easy to show that such a condition is not always satisfied when parental preferences are separable between consumption and fertility. Under a log-linear specification of preferences as in Kalemli-Ozcan [2003,2008], an additional birth reduces the parental loss coming from their risk aversion. Indeed, assume that $u(c_t, n_t) = \alpha \ln c_t + \gamma \ln n_t$. The first condition in proposition 1 reduces to $2 > 1$. Obviously, this condition is always satisfied. However, for $u(c_t, n_t) = v(c_t) + n_t^\beta + n_t(Bn_t + C)$ with $\beta \in]0, 1[$, $B < 0$ and $C > -[\beta\phi^{\beta-1} + \frac{2B}{\phi}]$, the first condition in proposition 1 is no more satisfied $\forall B < -\frac{\beta(\beta-1)^2}{2\phi^{\beta-2}}$ which means that parents undershoot their optimal number of children.²⁵

24. This term is simply equal to the variance of the binomial times a loss term coming from the risk aversion of parents since $u''_{nn} < 0$. Then, it is straightforward to calculate the impact of an additional birth on this loss:

$$\frac{\partial Loss}{\partial N} = \frac{q_t(1-q_t)N_t}{2} \left[\frac{u''_{nn_t}}{N_t} - (\phi + \theta e_t) w_t h_t u'''_{nnc_t} + q_t u'''_{nnn_t} \right] \quad (19)$$

Notice that this reasoning is not valid for non-separable utility functions out of the steady state. Following Leland [1968] and Kimball [1990], parents can be defined as prudent if they have a precautionary demand for children. It is clear that in an endogenous fertility problem, the usual condition $u''' > 0$ is not sufficient to observe prudent behavior. See for instance Langlais [1995] for a similar result about the risk on interest rate.

25. Indeed in this case, the overshooting condition reduces to $\beta(\beta-1)^2 n_t^{\beta-2} + 2B > 0$ what is not satisfied if $B < -\frac{\beta(\beta-1)^2}{2\phi^{\beta-2}}$.

It is also straightforward that the second condition of proposition 1 is not always satisfied. Once again, adopting a log-linear utility function implies that parents have a precautionary demand for children. However, adopting a utility function with a constant elasticity of substitution, the results can be completely reversed. Assume $u(c_t, n_t) = \left(C_t^{\frac{1}{\alpha}} + n_t^{\frac{1}{\alpha}}\right)^\alpha$ with $\alpha > 1$. It is straightforward that the condition can never be fulfilled $\forall \alpha > 2$.²⁶ In this case, the loss in term of utility due to risk aversion increases when parents decide to have an additional child.

In the present model, I assume that there is no externality. Intuitively, it should imply that the first best optimum is identical to the laissez-faire equilibrium. However this will be true only if the Social Planner is purely utilitarian and, when population is endogenous, assuming a purely utilitarian Social Planner is far from being the unique option.

3 The Social Optimum

When the size of the population is endogenous, defining the SWF is not straightforward. For instance, as shown by Nerlove and *al* [1986], adopting either a Millian SWF or a Benthamite SWF leads to very different recommendations of economic policy. In the present paper, I focus on "Millian-type" SWF in the sense that the Social Planner will always try to maximize individual utility rather than total utility.²⁷

However, under uncertain child survival, the problem of the "timing effect" has to be explored. I analyze two polar cases. First, the SWF coincides with the expected utility of the representative agent displayed in (9). In this case, the SWF is *ex-ante*. In other words, the Social Planner is purely utilitarian and so fully concerned with uncertainty at the individual level. Therefore, the SWF results from the aggregation of individual preferences which seems to be reasonable. In this case, overshooting as well as undershooting are not a source of inefficiency.

In the second polar case, the SWF consists of the *ex-post* SWF. In this case, the SWF doesn't equal the individual expected utility but the utility of the expected realization of

26. Indeed, after some calculus, this condition becomes $\forall t$:

$$C^{\frac{1}{\alpha}} \left[q + \frac{1}{\alpha} - 2 \right] + (qN)^{\frac{1}{\alpha}} \left[q - \frac{1}{\alpha} - 1 \right] - (\phi + \theta e) w h q N \left[\frac{1}{\alpha} \frac{C^{\frac{1}{\alpha}} + n^{\frac{1}{\alpha}}}{C} + \frac{\alpha - 2}{\alpha} q N^{\frac{1-\alpha}{\alpha}} \right] > 0 \quad (20)$$

This is impossible when $\alpha > 2$. Furthermore, it is straightforward that there exists $\bar{\alpha} < 2$ such that the second condition of proposition 1 isn't satisfied.

27. See Baudin [2010] for a discussion on the optimality properties of the Razin and Ben-Zion's model with non-Millian Social Welfare Functions and certainty about the parental reproductive success.

n_t . Because the law of large numbers does apply here, the Social Planner knows the average number of surviving children with almost certainty for each date t , and maximizes the utility derived from this realization. In other words, individual utility is evaluated after the resolution of uncertainty and the Social Planner can be considered as risk neutral. In this case, the individual precautionary demand for children is a source of inefficiency since all parents make more children than what is optimal without uncertainty, and so they don't allocate enough wealth to both consumption and education. In the same way, undershooting is also a source of inefficiency since parents allocate too much resource to both consumption and education.²⁸

Note that, in these two polar cases, the Social Planner is not concerned with inequalities and that nothing ensures that ex-post one of these SWF always provides more utility per se or more total utility than the other. I propose the following SWF that allows to represent these two polar cases. Let $W_{i,t}$ denote this SWF:

$$W_{i,t} = \sum_{t=0}^{\infty} \beta^t \left[u(c_t, q_t N_t) + \mu_i \frac{q_t(1-q_t)N_t}{2} u''_{n_t n_t}(c_t, q_t N_t) \right] \quad (21)$$

with $i = \{0, 1\}$. μ_i allows me to take the timing-effect into account. I assume $\mu_1 = 1$ what makes the SWF coincide with the *ex-ante* SWF. In this case, (21) is directly deduced from (9) using the same approximation as in the previous section. Alternatively, I assume $\mu_0 = 0$. It makes the SWF coincide with the *ex-post* SWF.^{29,30}

The resource constraint of the economy implies that total consumption equals total production such that:

$$L_t [c_t + \phi A h_t N_t + \theta A h_t N_t e_t] = A h_t L_t \quad (22)$$

28. Note that because child survival is subject to uncertainty, nothing ensures that the social optimum dominates the laissez-faire equilibrium in the sense of A and P efficiency (see Golosov *et al* [2007]). Indeed, even when the Social Planner is assumed to be ex-post, he cannot predict individual realization of risk. Furthermore, he or she has no instrument to ensure agents against risk since children are not an exchangeable good.

29. Gajdos and Maurin [2004] propose an axiomatic that defines a class of SWF which lie strictly between the *ex-ante* and the *ex-post* SWF. Here, nothing ensures that a SWF with μ_i taking intermediary values between zero and one would respect this axiomatic. For this reason, I don't explore the case of μ_i strictly lying between zero and one.

30. The consistency of notations would require to index all endogenous variables with subscripts i but I choose to delete them in order to make calculus more readable.

Let \mathcal{L}_0^S denote the Lagrangian of the problem, it equals:

$$\mathcal{L}_0^S = \sum_{t=0}^{+\infty} \beta^t \left[u(c_t, q_t N_t) + \mu_i \frac{q_t(1-q_t)N_t}{2} u''_{n_t n_t}(c_t, q_t N_t) + \eta_t (f(e_t, h_t) - h_{t+1}) \right] \quad (23)$$

where η_t denotes the Lagrange multiplier attached to the human capital accumulation process at date t . The Social Planner has to maximize (23) substituting c_t by its expression in (22). The first-order conditions with respect to N_t, e_t and h_{t+1} are:³¹

$$-(\phi + \theta e_t) A h_t \left(u'_{c_t} + \mu_i \frac{q_t(1-q_t)N_t}{2} u'''_{n_t n_t} \right) + q_t u'_{n_t} + \mu_i \frac{q_t(1-q_t)}{2} \left[u''_{n_t} + q_t N_t u'''_{n_t n_t} \right] = 0 \quad (24)$$

$$-\theta N_t A h_t \left(u'_{c_t} + \mu_i \frac{q_t(1-q_t)N_t}{2} u'''_{n_t n_t} \right) + \eta_t f'_1(e_t, \cdot) = 0 \quad (25)$$

$$A(1 - [\phi + \theta e_{t+1}] N_{t+1}) \left(u'_{c_{t+1}} + \mu_i \frac{q_{t+1}(1-q_{t+1})N_{t+1}}{2} u'''_{n_{t+1} n_{t+1}} \right) + \eta_{t+1} f'_2(e_{t+1}, \cdot) = \frac{\eta_t}{\beta} \quad (26)$$

Therefore, the Social Optimum is defined as the set $\{\hat{c}_t, \hat{N}_t, \hat{e}_t, \hat{h}_t, \hat{H}_t, \hat{Y}_t\}_{t=0}^{t=+\infty}$ satisfying equations $\{(5), (6), (22), (24), (25), (26)\}_{t=0}^{t=+\infty}$.³²

It is intuitive that the laissez-faire equilibrium coincides with the first best path when the SWF is *ex-ante*. However, when this latter is *ex-post*, decentralized fertility decisions are no more optimal since overshooting and undershooting are a source of social inefficiency.

4 On The Optimal Tax-Transfer Policy

To decentralize the social optimum, the government has to implement a public policy that makes the competitive equilibrium $\{c_t^*, N_t^*, e_t^*, h_t^*, H_t^*, Y_t^*\}_{t=0}^{t=+\infty}$ coincide with the social optimum $\{\hat{c}_t, \hat{N}_t, \hat{e}_t, \hat{h}_t, \hat{H}_t, \hat{Y}_t\}_{t=0}^{t=+\infty}$.

In this section, I characterize the optimal economic policy. I especially show that this optimal policy is unique whatever the SWF that is chosen. Then, I analyze the properties of this policy in the two polar cases: (i) the *ex-ante* SWF and (ii) the *ex-post* SWF. I show

31. Once again, to ensure global concavity of the problem, its Hessian Matrix is assumed to be negative semi-definite. Furthermore, the Social Planner's objective is bounded since $\lim_{T \rightarrow \infty} \beta^T \left[u(c_T, q_T N_T) + \mu_i \frac{q_T(1-q_T)N_T}{2} u''_{n_T n_T}(c_T, q_T N_T) \right] = 0$

32. The following transversality condition has also to be satisfied:

$$\lim_{t \rightarrow +\infty} \beta^t \frac{\theta N_t A h_t}{f'_1(e_t, h_t)} \left(u'_{c_t} + \mu_i \frac{q_t(1-q_t)N_t}{2} u'''_{n_t n_t} \right) h_{t+1} = 0 \quad (27)$$

that the nature of the optimal economic policy is completely different between these two cases.

Definition 2 Let the set $\{\lambda_t, \Lambda_t, T_t\}_{t=0}^{t=+\infty}$ define an economic policy where $\lambda_t > 0$ (resp $\lambda_t < 0$) consists of the subsidy rate (resp tax rate) on education spending, $\Lambda_t > 0$ (resp $\Lambda_t < 0$) denotes the subsidy (resp tax) on each child birth and $T_t \gtrless 0$ a lump sum transfer.

At each date t , the government budget constraint has to be balanced such that:³³

$$T_t = \lambda_t \theta e (h_{t+1}^*, h_t^*) N_t^* A h_t^* + \Lambda_t N_t^* A h_t^* \quad (28)$$

The parental budget constraint is now:

$$c_t = w_t h_t (1 - [\phi - \Lambda_t + (1 - \lambda_t) \theta e_t] N_t) + T_t \quad (29)$$

The competitive equilibrium is now defined as the set $\{c_t^*, N_t^*, e_t^*, h_t^*, H_t^*, Y_t^*, w_t^*\}_{t=0}^{t=+\infty}$ satisfying equations $\{(5), (6), (7), (29)\}_{t=0}^{t=+\infty}$ and the following first-order conditions with respect to N_t, e_t and h_{t+1} .³⁴

$$-(\phi - \Lambda_t + (1 - \lambda_t) \theta e_t) w_t h_t \left(u'_{c_t} + \frac{q_t(1-q_t)N_t}{2} u'''_{nnc_t} \right) + q_t u'_{n_t} + \frac{q_t(1-q_t)}{2} \left[u''_{nn_t} + q_t N_t u'''_{nnn_t} \right] = 0 \quad (31)$$

$$-(1 - \lambda_t) \theta N_t w_t h_t \left(u'_{c_t} + \frac{q_t(1-q_t)N_t}{2} u'''_{nnc_t} \right) + \eta_t f'_1(e_t, \cdot) = 0 \quad (32)$$

$$w_{t+1} [1 - (\phi - \Lambda_{t+1} + (1 - \lambda_{t+1}) \theta e_{t+1}) N_{t+1}] \left(u'_{c_{t+1}} + \frac{q_{t+1}(1-q_{t+1})N_{t+1}}{2} u'''_{nnc_{t+1}} \right) + \eta_{t+1} f'_2(e_{t+1}, \cdot) = \frac{\eta_t}{\beta} \quad (33)$$

An optimal economic policy has to make the set $\{(24), (25), (26)\}_{t=0}^{t=+\infty}$ identical to the set $\{(31), (32), (33)\}_{t=0}^{t=+\infty}$. Indeed, the combination of (28) and (29) ensures that the resource constraint is satisfied.

Proposition 2 Given that the Social Planner can observe the parental rational expectations on the future tax and transfers $\{\lambda_{t+1}, \Lambda_{t+1}, T_{t+1}\}_{t=0}^{t=+\infty}$, there exists a unique economic policy $\{\hat{\lambda}_t, \hat{\Lambda}_t, \hat{T}_t\}_{t=0}^{t=+\infty}$ that is able to decentralize the first-best path for each μ_i :

33. Notice that, in this paper, family policies are limited to tax - transfer policies. In reality, family policies include a large set of instruments like, for instance, coercive policies and preventive actions.

34. The transversality condition becomes:

$$\lim_{t \rightarrow +\infty} \beta^t \frac{(1 - \lambda_t) \theta N_t A h_t}{f'_1(e_t, h_t)} \left(u'_{c_t} + \frac{q_t(1-q_t)N_t}{2} u'''_{nnc_t} \right) h_{t+1} = 0 \quad (30)$$

- If the Social Planner is purely utilitarian ($\mu_i = 1$), the laissez-faire equilibrium coincides with the first best path and no economic policy is required such that $\{\hat{\lambda}_t, \hat{\Lambda}_t, \hat{T}_t\}_{t=0}^{t=+\infty} = \{0, 0, 0\}$.
- If the Social Planner is ex-post ($\mu_i = 0$), the optimal economic policy is described as follows:

$$\hat{\lambda}_t = \mathcal{A}_t \hat{\lambda}_{t+1} + \mathcal{G}_t (\mu_i - 1) \quad (34)$$

$$\hat{\Lambda}_t = -\mathcal{J}_t \hat{\lambda}_{t+1} + \mathcal{R}_t (\mu_i - 1) \quad (35)$$

The optimal value \hat{T}_t is directly deduced from the government budget constraint.

Proof. See Appendix B. ■

This result shows that it is possible to decentralize the social optimum all along the transition to the steady state even when the number of children surviving to adulthood is uncertain. In accordance with the intuition, no economic policy is required when the Social Planner is purely utilitarian. Indeed, with an *ex-ante* SWF, there exist no difference between private and social preferences. Then, it is straightforward that, in absence of any externality in the model, the first best and the competitive equilibrium coincide.

However, when the Social Planner is *ex-post*, the optimum and the competitive equilibrium no more coincide. Indeed, in this case, the existence of a precautionary demand for children as well as undershooting are a source of social inefficiency. The stationary values of the instruments are the following:

$$\hat{\lambda} = \frac{\beta}{1 - \epsilon_2^f} \cdot \frac{q(1-q)N^2\epsilon_1^f}{2N\theta e} \cdot \frac{\frac{u_{nn}''}{N} - (\phi + \theta e)Ah u_{nnc}''' + q u_{nnn}'''}{Ah \left(u_c' + \frac{q(1-q)N}{2} u_{nnc}''' \right)} \quad (36)$$

$$\hat{\Lambda} = -\frac{q(1-q)N}{2} \cdot \frac{\frac{u_{nn}''}{N} - (\phi + \theta e)Ah u_{nnc}''' + q u_{nnn}'''}{Ah \left[u_c' + \frac{q(1-q)N}{2} u_{nnc}''' \right]} \cdot \frac{1 + \frac{\beta}{N} - \epsilon_2^f}{1 - \epsilon_2^f} \quad (37)$$

Remembering Proposition 1, $\hat{\Lambda} < 0$ while $\hat{\lambda} > 0$ when parents overshoot their number of children.³⁵ Indeed, hoarding is now a source of inefficiency: risk aversion leads parents to have more children than optimal. Then, births must be taxed and because the parental budget constraint is non-linear, the government has to subsidize education. Indeed, when parents invest in their children's human capital, they increase the future quantity cost of grandchildren that is $\phi w_{t+1} h_{t+1} N_{t+1}$ what finally lowers the returns to educational investment.

35. ϵ_x^b denotes the elasticity of b with respect to x . $\epsilon_2^f < 1$ by assumption

When the tax on births is implemented, the cost becomes $(\phi - \Lambda_{t+1})w_{t+1}h_{t+1}N_{t+1}$ with $\Lambda_{t+1} < 0$. Then the returns to the investment in education is decreased below its optimal level and so, it is optimal to subsidize education in order to compensate for this distortion created by the economic policy.

Conversely, if parents undershoot their optimal number of children, $\hat{\Lambda} > 0$ while $\hat{\lambda} < 0$. In this case, parents protect themselves from risk by having less children than socially optimal. The government has to subsidize births and because the parental budget constraint is not linear, education has to be taxed.

It is intuitive that this result becomes crucial when positive externalities are introduced in the process of human capital accumulation. Indeed, in the case of an *ex-post* SWF, the effect of undershooting could dominate the usual impact of positive externalities of education. The next section discusses this issue.

5 A Simple Extension To Educational Externalities

A large majority of the literature assumes that there exist some externalities in the production of human capital. Introducing this kind of externalities is crucial to discuss the optimality conditions in the models of trade-off between quality and quantity. Intuitively, the existence of uncertainty about child survival to adulthood could be crucial to determine the nature of the optimal economic policies as well as the optimal response to a mortality crisis. Human capital is now produced thanks to the following process:

$$h_{t+1} = f(e_t, h_t, \bar{h}_t) \quad (38)$$

Function f is strictly increasing and concave with respect to all its arguments, it is close to the De la Croix and Doepke [2003]'s production function of human capital. I assume non increasing returns to scale. In addition to previous inputs, there exists a Lucas' type aggregate externality in the sense that the average level of human capital in the population \bar{h}_t has a positive impact on the children's future human capital. This assumption is in line, among others, with Lucas [1988] and De la Croix and Doepke [2003]. Therefore, parents don't internalize that their children's human capital affects the production of human capital of other people's grandchildren ($h_{t+2} = f[e_{t+1}, h_{t+1}, \bar{h}_{t+1}]$).

I use exactly the same method as in sections 3 to 6 in order to determine the competitive equilibrium, the first best path and the optimal economic policy. It is straightforward that the only difference between the alternative cases is that the Social Planner takes into account

all the returns of his investment in human capital while parents don't take into account the Lucas type externality. Then *ceteris paribus*, parents should tend to underinvest in their children's quality.

Proposition 3 *Given that the Social Planner can observe the parental rational expectations on the future tax and transfers $\{\lambda_{t+1}, \Lambda_{t+1}, T_{t+1}\}_{t=0}^{t=+\infty}$, there exists a unique economic policy $\{\hat{\lambda}_t, \hat{\Lambda}_t, \hat{T}_t\}_{t=0}^{t=+\infty}$ that is able to decentralize the first-best path whatever the risk aversion of the Social Planner. This optimal economic policy is described as follows:*

$$\hat{\lambda}_t = \mathcal{A}_t \hat{\lambda}_{t+1} + \mathcal{D}_t \epsilon_3^{f(e_{t+1}, h_t, \bar{h}_t)} + \mathcal{G}_t (\mu_i - 1) \quad (39)$$

$$\hat{\Lambda}_t = -\mathcal{J}_t \hat{\lambda}_{t+1} - \mathcal{L}_t \epsilon_3^{f(e_{t+1}, h_t, \bar{h}_t)} + \mathcal{R}_t (\mu_i - 1) \quad (40)$$

The optimal value \hat{T}_t is directly deduced from the government budget constraint.

Proof. See Appendix C. ■

The stationary values of the instruments are the following:

$$\hat{\lambda} = \frac{\beta}{1 - \epsilon_2^f} \left[\epsilon_3^f + (1 - \mu_i) \frac{q(1-q)N\epsilon_1^f \frac{u''_{nn}}{N} - (\phi + \theta e)whu'''_{nnc} + qu'''_{nnn}}{2\theta e} \frac{1}{Ah \left(u'_c + \frac{q(1-q)N}{2} u'''_{nnc} \right)} \right] \quad (41)$$

$$\hat{\Lambda} = -\frac{\beta\theta e\epsilon_3^f}{1 - \epsilon_2^f} - (1 - \mu_i) \frac{q(1-q)N}{2} \cdot \frac{\frac{u''_{nn}}{N} - (\phi + \theta e)Ah u'''_{nnc} + qu'''_{nnn}}{Ah \left[u'_c + \frac{q(1-q)N}{2} u'''_{nnc} \right]} \cdot \frac{1 + \frac{\beta}{N} - \epsilon_2^f}{1 - \epsilon_2^f} \quad (42)$$

In the following subsections, I discuss the properties of this optimal economic policy at the steady state in the two polar cases: *ex-ante* and *ex-post* SWF.

5.1 *Ex-Ante* Social Welfare Function, $\mu_i = 1$

Once again, when the Social Planner is purely utilitarian, there is no difference between the individual welfare function and the SWF and so, uncertainty and risk aversion has no role to play in the design of the optimal economic policy.³⁶ At the steady state, the optimal values of the instruments when $\mu_i = 1$, are the following:

$$\hat{\lambda} = \frac{\beta\epsilon_3^f}{1 - \epsilon_2^f} > 0, \quad \hat{\Lambda} = \frac{-\beta\theta e\epsilon_3^f}{1 - \epsilon_2^f} < 0 \quad (43)$$

36. This is in line with the first case in Proposition 2. Obviously, because the degree of risk-aversion has an impact on the optimal values of individual and social choices, it has an impact on the magnitude of both $\hat{\lambda}$ and $\hat{\Lambda}$ but not on their sign.

The optimal economic policy consists of subsidizing the parental investments in education and taxing births. This result crucially comes from both the Lucas-type externality and the non-linearity of the parental budget constraint.³⁷ Because of the Lucas-type externality, parents don't take into account all the returns of their investment in their children's human capital. Therefore, they tend to underinvest in the quality of children. This inefficiency has to be corrected by the implementation of a subsidy on education. However, the subsidy on education spending also reduces the marginal cost of the quantity of children that becomes $\phi w_t h_t + (1 - \lambda_t)\theta w_t h_t e_t$. It implies that parents have too much children.³⁸ This has to be corrected by the implementation of a tax on each child birth. The non-linearity of the parental budget constraint implies that three instruments are needed to correct the only Lucas-type externality.

Importantly, the nature of the optimal economic policy won't be modified by the apparition of a mortality crisis. When a mortality crisis appears,³⁹ parents engage either in overshooting or in undershooting what is socially optimal. Then, the only difference between the first best and the competitive equilibrium still consists in the Lucas type externality which requires to subsidize education and to tax births.

5.2 *Ex-Post* Social Welfare Function, $\mu_i = 0$

With an ex-post SWF, two main effects interact: (i) the Lucas type externality that makes parents underinvest in their children's human capital and (ii) the undershooting (resp. overshooting) that makes parents having too little (resp. too many) children at the competitive equilibrium. Therefore, the optimal economic policy will be determined by the respective magnitude of these two effects. Formally, the optimal values of $(\hat{\lambda}, \hat{\Lambda})$ are displayed in equations (41) and (42) with $\mu_i = 0$.

37. It generalizes Baudin [2010] who finds the same result in a Millian framework without uncertainty on the child survival to adulthood.

38. Parents could also have a too low number of children if after the reduction in the cost of both quality and quantity, they reduce their fertility. See Willis [1973] and Jones and Schoonbroodt [2007] for a discussion of this point.

39. Assume that q initially equals one and that, after the mortality shock, it becomes smaller than one.

Definition 3

$$\mathcal{Z} \equiv -\frac{q(1-q)N}{2} \frac{1 + \frac{\beta}{N} - \epsilon_2^f \frac{u_{nn}''}{N} - (\phi + \theta e) Ah u_{nnc}''' + q u_{nnn}'''}{\beta \theta e Ah \left[u_c' + \frac{q(1-q)N}{2} u_{nnc}''' \right]}$$

$$\mathcal{W} \equiv -\frac{q(1-q)N \epsilon_1^f \frac{u_{nn}''}{N} - (\phi + \theta e) Ah u_{nnc}''' + q_t u_{nnn}'''}{2 \theta e Ah \left[u_c' + \frac{q(1-q)N}{2} u_{nnc}''' \right]}$$

Proposition 4 *When parents formulate a precautionary demand for children, the optimal economic policy always consists of taxing births and subsidizing education. When parents undershoot their number of children, the optimal economic policy consists of:*

- *a subsidy on education and a tax on births when $\epsilon_3^f > \mathcal{Z}$*
- *a subsidy on both education and births when $\epsilon_3^f \in [\mathcal{W}, \mathcal{Z}]$*
- *a tax on education and a subsidy on births when $\epsilon_3^f < \mathcal{W}$*

Proof. See Appendix D. ■

The interpretation of this result is very intuitive. When parents overshoot their optimal number of children, the two main effects reinforce each other: (i) because of the Lucas type externality, education has to be subsidized and births have to be taxed, (ii) because the parental precautionary demand for children is a source of inefficiency, births must be taxed and education subsidized. Therefore, when parents have a precautionary demand for children, it is always optimal to subsidize education and to tax births.

However, when parents undershoot their number of children, the nature of the optimal economic policy can be radically different. Indeed, the two main effects go in the opposite direction: (i) because of the Lucas type externality, education has to be subsidized and births have to be taxed while (ii) because risk averse parents protect themselves from risk by having less children than socially optimal, births have to be subsidized and education must be taxed. As a result, if the Lucas-type externality is weak relative to the "undershooting effect" ($\epsilon_3^f < \mathcal{W}$), it is optimal to subsidize births and to tax education. For intermediary intensities of the Lucas-type externality ($\epsilon_3^f \in [\mathcal{W}, \mathcal{Z}]$), the optimal economic policy consists of subsidizing both births and education. Obviously, for strong intensities of the externality ($\epsilon_3^f > \mathcal{Z}$), it is optimal to subsidize education and to tax births as in the case of overshooting.

With an *ex-post* SWF, the nature of the optimal economic policy can be profoundly altered by the apparition of a mortality crisis. This can easily be understood in the light

of a comparison between the optimal economic policies when $q = 1$ and when $q < 1$.⁴⁰ When the mortality rate is zero ($q = 1$), parents behave as in a certain environment and the optimal economic policy consists of subsidizing education and taxing births because of the Lucas-type externality (see equations (36) and (37)). When the mortality crisis does appear, parents can either engage in overshooting or undershooting what is socially inefficient.

On the one hand, if parents engage in overshooting, the need to tax births and to subsidize education is reinforced.⁴¹ On the other hand, if parents undershoot their number of children, the optimal government's response can be more surprising. Indeed, if parents *strongly* undershoot their fertility when the mortality crisis takes place, the "undershooting effect" can fully dominate the Lucas type externality. It would imply that education has to be taxed and births must be subsidized. This case arises only if, after the decrease in q , $\epsilon_3^f < \mathcal{W}$. If the "undershooting effect" is less intense ($\epsilon_3^f \in [\mathcal{W}, \mathcal{Z}]$), the optimal modification of the economic policy is to subsidize births instead of taxing it and to keep subsidizing educational investments. Indeed, in this case, the "undershooting effect" doesn't fully dominate the Lucas-type externality.⁴²

6 Conclusion

The present paper investigates the impact of uncertain child survival to adulthood on both individual fertility rates and the optimal economic policy. I especially show that facing this uncertainty, parents don't always formulate a precautionary demand for children. I also show that overshooting as well as undershooting become a source of inefficiency when social welfare is evaluated after the resolution of uncertainty. It is then optimal to tax births and to subsidize investments in education when parents overshoot their fertility rate. Conversely,

40. As mentioned by Kalemli-Ozcan [2003], mortality rates never go beyond one half that is the mortality rate implying the highest variance in the number of surviving children.

41. Obviously, the net impact of this change on both $\hat{\Lambda}$ and $\hat{\lambda}$ depends on the modification of the parental trade-off between quality and quantity when q decreases:

$$\frac{d\hat{\Lambda}}{dq} = \frac{\beta\theta\epsilon_3^f \frac{de}{dq}}{1 - \epsilon_2^f} + \frac{\partial}{\partial q} \left[\frac{q(1-q)N}{2} \frac{u_{nnn}'' - (\phi + \theta e)Ah u_{nnc}''' + q u_{nnn}'''}{Ah \left[u_c' + \frac{q(1-q)N}{2} u_{nnc}''' \right]} \frac{1 + \frac{\beta}{N} - \epsilon_2^f}{1 - \epsilon_2^f} \right] \quad (44)$$

Nevertheless, $\hat{\Lambda}$ will always remain negative and $\hat{\lambda}$ positive.

42. In this section, I discuss the impact of the emergence of a mortality crisis in two polar cases: the *ex-ante* SWF and the *ex-post* SWF. Obviously, admitting that the SWF is either strictly *ex-ante* or strictly *ex-post* is a simplification of a more general case where it is a mix of these two polar cases. Intuitively, in this case, the existence of either a precautionary demand for children or undershooting would remain a source of inefficiency that has to be corrected thanks to distortive taxes.

it becomes optimal to subsidize births and to tax education when parents undershoot their fertility rate. Introducing positive externalities in the accumulation of human capital can partially alter this result.

To extend these results, future research should explore two problems: (i) the definition of conditions under which parents have a precautionary demand for children out of the steady state in a framework where their utility function is non-separable and (ii) the consideration of the old-age support motive for child births in developing countries.

References

- [1] Baland J.M. and Estevan F. [2007] "Mortality risks, education and child labor", *Journal of Development Economics*, vol. 84(1), p118-137
- [2] Bar M. & Leukhina O. [2010] "Demographic Transition and Industrial Revolution: A Macroeconomic Investigation", *Review of Economic Dynamics*, vol. 13(2), p 424-451
- [3] Baudin T. [2010] "Family Policies: What Does The Standard Endogenous Fertility Model tell Us?", *Journal of Public Economic Theory*, forthcoming
- [4] Becker G.S. [1973] "A Theory of Marriage: Part I", *Journal of Political Economy*, vol. 81(4), p 813-46
- [5] Becker G.S and Barro R.J. [1988] "A Reformulation of the Economic Theory of Fertility", *The Quarterly Journal of Economics*, MIT Press, vol. 103(1), p 1-25, February
- [6] Becker G.S. and Lewis H.G. [1973] "On the Interaction Between the Quantity and Quality of Children", *The Journal of Political Economy*, vol. 81(2) Part 2 New Economic Approach to Fertility, p 279-288
- [7] Becker G.S. and Tomes N. [1976] "Child Endowments and the Quantity and Quality of Children", *The Journal of Political Economy*, vol. 84 (4) Part 2, Essays in Labor Economics in Honor of H.G. Lewis, p 143-162
- [8] Bell C. and Gersbach H. [2009] "The macroeconomics of targeting: the case of an enduring epidemic", *Journal of Health Economics*, vol. 28(1), p54-72
- [9] Boerma J.T., Nunn A.J. Whitworth J.A.G. [1998] "Mortality impact of the AIDS epidemic: evidence from community studies in less developed countries" *AIDS* vol. 12(1), p S3-S14

- [10] Boucekkine R., De la Croix D. and Licandro O. [2003] "Early Mortality Declines at the Dawn of Modern Growth", *Scandinavian Journal of Economics*, vol. 105(3), p401-418
- [11] Boucekkine R., Desbordes R. and Latzer H. [2009] "How do epidemics induce behavioral changes?", *Journal of Economic Growth*, vol. 14(3), p233-264
- [12] Buthelezi T., Velia M. Yamauchi F. [2008] "Impacts of Prime-age Adult Mortality on Labour Supply: Evidence from Adolescents and Women in South Africa" *Oxford Bulletin of Economics and Statistics*, vol. 70(3), p 375-398
- [13] Cervellati M. and Sunde U. [2007] "Human Capital, Mortality and Fertility: A Unified Theory of the Economic and Demographic Transition", *CEPR Discussion Papers 6384*, C.E.P.R. Discussion Papers
- [14] Cervellati M. and Sunde U. [2009] "Life Expectancy and Economic Growth: The Role of the Demographic Transition", *CEPR Discussion Papers 7361*, C.E.P.R. Discussion Papers
- [15] Chakraborty S. [2004] "Endogenous Lifetime and Economic Growth", *Journal of Economic Theory*, vol. 116(1), p 119-137
- [16] De La Croix D. and Doepke M. [2003] "Inequality and Growth: Why Differential Fertility Matters?", *American Economic Review*, American Economic Association, vol. 93(4), p 1091-1113
- [17] Doepke M. [2005] "Child mortality and fertility decline: Does the Barro-Becker model fit the facts?", *Journal of Population Economics*, vol. 18(2), p 337-366
- [18] Eckstein Z. and Wolpin K.I. [1985] "Endogenous fertility and optimal population size", *Journal of Public Economics*, vol. 27(1), p 93-106
- [19] Ehrlich I. and Lui F.T. [1991] "Intergenerational Trade, Longevity, and Economic Growth", *Journal of Political Economy*, vol 99(5), p1029-59
- [20] Feeney G. [2001] "The Impact of HIV/AIDS on Adult Mortality in Zimbabwe" *Population and Development Review*, vol. 27(4), p 771-780.
- [21] Gajdos T. and Maurin E. [2004] "Unequal uncertainties and uncertain inequalities: an axiomatic approach", *Journal of Economic Theory*, vol. 116(1), p 93-118

- [22] Galor O. and Weil D.N [1999]. "From Malthusian stagnation to modern growth", *American Economic Review*, vol. 89, p150-154
- [23] Galor O. [2005] "From Stagnation to Growth: Unified Growth Theory", in: Philippe Aghion and Steven Durlauf (ed.), *Handbook of Economic Growth*, edition1, vol 1, chapter 4, p171-293 Elsevier
- [24] Golosov M., Jones L.E. & Tertilt M [2007] "Efficiency with Endogenous Population Growth," *Econometrica*, Econometric Society, vol. 75(4), p1039-1071
- [25] Jones L. E. & Schoonbroodt A. [2007] "Complements versus Substitutes and Trends in Fertility Choice in Dynastic Models", *NBER Working Paper 13680*, National Bureau of Economic Research
- [26] Jones L. E. & Schoonbroodt A. [2009] "Complements versus Substitutes and Trends in Fertility Choice in Dynastic Models: Additional Appendix", Working paper, University of Southampton, available at <http://www.economics.soton.ac.uk/staff/alicesch/research.html>
- [27] Kalemli-Ozcan S. [2003] "A stochastic Model of Mortality, Fertility and Human Capital Investment", *Journal of Development Economics*, vol. 70(1), p 103-118
- [28] Kalemli-Ozcan S. [2008] "The uncertain lifetime and the timing of human capital investment", *Journal of Population Economics*, vol. 21(3), p557-572
- [29] Kimball M.S. [1990] "Precautionary Saving in the Small and in the Large" *Econometrica*, vol 58(1), p53-73
- [30] Lagerlof N.P. [2003] "From Malthus to modern growth: can epidemics explain the three regimes?", *International Economic Review*, vol. 44, p. 755-777
- [31] Langlais E. [1995] "Aversion au risque et prudence : le cas d'un risque de taux d'intérêt" *Revue Economique*, vol 46(4), p1099-1119
- [32] Leland H.E. [1968] "Saving and Uncertainty: The Precautionary Demand for Saving" *The Quarterly Journal of Economics*, Vol. 82(3), p465-473
- [33] Lopez A.D. Murray C.J. [1997] "Global mortality, disability, and the contribution of risk factors: Global Burden of Disease Study" *The Lancet* 12;350(9071):144

- [34] Lucas R.J. [1988] "On the mechanics of economic development", *Journal of Monetary Economics*, vol. 22(1), p 3-42
- [35] Meslé F. and Shkolnikov V.M. [1996] "Russian epidemiological crisis as mirrored by mortality trends", In: Julie DaVanzo ed. *Russia's Demographic Crisis*, RAND, p113-162
- [36] Myerson R.B. [1981] "Utilitarianism, Egalitarianism, and the Timing Effect in Social Choice Problems", *Econometrica*, vol. 49(4), p 883-97
- [37] Nerlove M., Razin A. and Sadka E. [1986] "Some Welfare Theoretic Implications of Endogenous Fertility", *International Economic Review*, vol 27(1), p3-31
- [38] Nerlove M. & Raut L.K. [1997] "Growth models with endogenous population: A general framework", *Handbook of Population and Family Economics*, in: M. R. Rosenzweig & Stark, O. (ed.), edition 1, volume 1, chapter 20, p 1117-1174
- [39] Razin A. and Ben-Zion U. [1975] "An Intergenerational Model of Population Growth", *American Economic Review*, vol. 65(5), p 923-33
- [40] Sah R.K. [1991] "The Effect of Child Mortality Changes on Fertility Choice and Parental Welfare", *The Journal of Political Economy*, vol. 99(3), p 582-606
- [41] Sandmo A. [1983] "*ex post* Welfare Economics and the Theory of Merit Goods", *Economica*, vol. 50(197), p 19-33
- [42] Soares R. [2005] "Mortality Reductions, Educational Attainment, and Fertility Choice", *American Economic Review*, vol. 95(3), p580-601.
- [43] Spiegel [1993] "Rawlsian optimal population size", *Journal of Population Economics*, vol.3, p 363-373.
- [44] Strulik H. [2004] "Economic Growth and Stagnation with Endogenous Health and Fertility", *Journal of Population Economics*, vol. 17(3), p433-453
- [45] Willis R.J. [1973] "A New Approach To The Economic Theory of Fertility Behavior", *The Journal of Political Economy*, vol. 81(2), S14-S64
- [46] Zhang J. and Zhang J. [2006] "The Effect of Life Expectancy on Fertility, Saving, Schooling and Economic Growth: Theory and Evidence", *Scandinavian Journal of Economics*, vol. 107(1), p45-66

Appendix A

Separable preferences

From equations (16) and (17), it is straightforward that under the assumption of separable preferences, the first order conditions with respect to e_t and h_{t+1} are not affected by uncertainty. Therefore, the marginal net utility of an additional birth for a parent born in $t - 1$ can easily be calculated by differentiating (14) with respect to N_t . This yields:

$$\frac{\partial \mathcal{L}_t}{\partial N_t} = -(\phi + \theta e_t)w_t h_t u'_{c_t} + q_t u'_{n_t} + \frac{q_t(1 - q_t)}{2} \left[u''_{nn_t} + \frac{q_t N_t u'''_{nnn_t}}{2} \right] \quad (.45)$$

When parents determine their optimal fertility rate, $\frac{\partial \mathcal{L}_t}{\partial N_t} = 0$. Let N^u denote this optimal fertility rate. Parents will have a precautionary demand for children if their fertility rate is higher than what would be optimal in an environment without uncertainty. In such an environment, the optimal fertility rate N^{rl} solves $\frac{\partial \mathcal{L}_t^{rl}}{\partial N_t} = -(\phi + \theta e_t)w_t h_t u'_{c_t} + q_t u'_{n_t} = 0$.⁴³ Recalling that both problems are globally concave, if $\frac{\partial \mathcal{L}_t}{\partial N_t} < 0$, parents have more children than optimal and if it is positive, they have less children than optimal. From (.45), it follows that:

$$\frac{\partial \mathcal{L}_t}{\partial N_t} = \frac{\partial \mathcal{L}_t^{rl}}{\partial N_t} + \mathcal{X}_t \quad (.46)$$

with $\mathcal{X}_t \equiv \frac{q_t(1-q_t)}{2} \left[u''_{nn_t} + \frac{q_t N_t u'''_{nnn_t}}{2} \right]$.

This implies that $\frac{\partial \mathcal{L}_t}{\partial N_t} = 0$ if and only if $\frac{\partial \mathcal{L}_t^{rl}}{\partial N_t} = -\mathcal{X}_t$. When $\mathcal{X}_t > 0$, $\frac{\partial \mathcal{L}_t^{rl}}{\partial N_t} < 0$ and so $N^u > N^{rl}$: parents have a precautionary demand for children. Conversely, if $\mathcal{X}_t < 0$, $N^u < N^{rl}$ meaning that parents undershoot their optimal number of children.

Steady State

The combination of the first order conditions with respect to e_t and h_{t+1} implies that the optimal decisions of parents on (N_t, e_t) have to satisfy (15) at the steady state and the following condition:

$$1 - \phi N - \theta N e + \frac{\theta N h f'_2(e, \cdot, \cdot)}{\beta f'_1(e, \cdot, \cdot)} = 0 \quad (.47)$$

Finally, this condition doesn't depend on the parental risk aversion. Then, using the same method as for proposition 1, I obtain that the marginal net utility of an additional birth

43. Where \mathcal{L}_t^{rl} denotes the Lagrangian of the parental maximization problem without uncertainty.

when child survival is uncertain, is:

$$\frac{\partial \mathcal{L}_t}{\partial N_t} = -(\phi + \theta e_t) w_t h_t \left[u'_{c_t} + \frac{q_t(1-q_t)N_t}{2} u'''_{nnc_t} \right] + q_t u'_{n_t} + \frac{q_t(1-q_t)}{2} \left[u''_{nn_t} + \frac{q_t N_t u''''_{nnn_t}}{2} \right] \quad (.48)$$

Let $\mathcal{T} \equiv \frac{q(1-q)N}{2} \left[\frac{u''_{nn}}{N} - (\phi + \theta e) w h u'''_{nnc} + u'''_{nnn} \right]$. Using the same method as in the previous case, it is straightforward that parents have a precautionary demand for children when $\mathcal{T} > 0$ that is to say when $\frac{u''_{nn}}{N} > (\phi + \theta e) w h u'''_{nnc} - u'''_{nnn}$. Parents undershoot their optimal number of children otherwise.

Appendix B

Definition 4

$$\underline{\alpha}_t \equiv \frac{q_t(1-q_t)}{2} u''_{nn_t} + \frac{q_t^2(1-q_t)N_t}{2} u'''_{nnn_t} \quad (.49)$$

$$\underline{\omega}_t \equiv \frac{q_t(1-q_t)N_t}{2} u'''_{nnc_t} \quad (.50)$$

Analyzing sub-systems $\{(31), (32), (33)\}_{t=0}^{t=+\infty}$ and $\{(24), (25), (26)\}_{t=0}^{t=+\infty}$, it is straightforward that the set $\{\hat{\lambda}_t, \hat{\Lambda}_t\}_{t=0}^{t=+\infty}$ has to ensure that at each date t , the first order conditions with respect to N_t, e_t and h_{t+1} at the competitive equilibrium are identical to the these at the social optimum. This is ensured if the following system is satisfied at each date t :

$$-(\phi - \Lambda_t + [1 - \lambda_t] \theta e_t) A h_t [u'_{c_t} + \underline{\omega}_t] + q_t u'_{n_t} + \underline{\alpha}_t = -(\phi + \theta e_t) A h_t [u'_{c_t} + \mu_i \underline{\omega}_t] + q_t u'_{n_t} + \mu_i \underline{\alpha}_t \quad (.51)$$

$$\begin{aligned} & \left(1 - [\phi - \Lambda_{t+1}] N_{t+1} - [1 - \lambda_{t+1}] \theta N_{t+1} e_{t+1} + \frac{[1 - \lambda_{t+1}] \theta N_{t+1} h_{t+1} f'_2(e_{t+1}, \cdot, \cdot)}{\beta f'_1(e_{t+1}, \cdot, \cdot)} \right) \frac{u'_{c_{t+1}} + \underline{\omega}_{t+1}}{h_t} + \frac{[1 - \lambda_t] \theta N_t}{\beta f'_1(e_t, \cdot, \cdot)} [u'_{c_t} + \underline{\omega}_t] = \\ & - \frac{\theta N_t}{\beta f'_1(e_t, \cdot, \cdot)} [u'_{c_t} + \mu_i \underline{\omega}_t] + \left[1 - \phi N_{t+1} - \theta N_{t+1} e_{t+1} + \frac{\theta N_{t+1} h_{t+1} (f'_2(e_{t+1}, \cdot, \cdot) + f'_3(e_{t+1}, \cdot, \cdot))}{\beta f'_1(e_{t+1}, \cdot, \cdot)} \right] \frac{u'_{c_{t+1}} + \mu_i \underline{\omega}_{t+1}}{h_t} \end{aligned} \quad (.52)$$

Equations (.51) and (.52) characterize a system of two equations with two unknowns which are $\{\lambda_t, \Lambda_t\}$ given the parental rational expectation on $\{\lambda_{t+1}, \Lambda_{t+1}\}$. This system is linear with regards to its unknowns and so, it is straightforward that it admits a unique solution. Two cases have to be considered: (i) $\mu_i = 1$ and (ii) $\mu_i = 0$.

- $\mu_i = 1$: It is straightforward that the solution to the system is $\{\lambda_t, \Lambda_t\} = \{0, 0\}$. This is satisfied $\forall t$.

- $\mu_i = 0$: I can display $\{\lambda_t, \Lambda_t\}$ as linear functions of λ_{t+1} and Λ_{t+1} such that:

$$\widehat{\lambda}_t = \mathcal{A}_t \widehat{\lambda}_{t+1} + \mathcal{G}_t (\mu_i - 1) \quad (.53)$$

$$\widehat{\Lambda}_t = -\mathcal{J}_t \widehat{\lambda}_{t+1} + \mathcal{R}_t (\mu_i - 1) \quad (.54)$$

with

$$\begin{aligned} \mathcal{A}_t &\equiv \frac{\beta N_{t+1} h_{t+1} (u'_{c_{t+1}} + \omega_{t+1}) f'_2(e_{t+1}, \cdot) f'_1(e_t, \cdot)}{N_t h_t (u'_{c_t} + \omega_t) f'_1(e_{t+1}, \cdot)} \\ \mathcal{G}_t &\equiv \left(\frac{\beta f'_1(e_t, \cdot)}{\theta N_t h_t (u'_{c_t} + \omega_t)} + \frac{\beta N_{t+1} h_{t+1} f'_2(e_{t+1}, \cdot) f'_1(e_t, \cdot)}{N_t h_t (u'_{c_t} + \omega_t) f'_1(e_{t+1}, \cdot)} \right) \omega_{t+1} - \frac{\omega_t}{u'_{c_t} + \omega_t} - \frac{\beta f'_1(e_t, \cdot) N_{t+1} \alpha_{t+1}}{A h_{t+1} \theta N_t h_t (u'_{c_t} + \omega_t)} \\ \mathcal{J}_t &\equiv -\theta e_t \mathcal{A}_t \\ \mathcal{R}_t &\equiv \frac{\alpha_t - (\phi + \theta e_t) A h_t \omega_t}{A h_t (u'_{c_t} + \omega_t)} - \frac{\beta e_t f'_1(e_t, \cdot) \omega_{t+1}}{N_t h_t (u'_{c_t} + \omega_t)} \left(1 + \frac{\theta N_{t+1} h_{t+1} f'_2(e_{t+1}, \cdot)}{f'_1(e_{t+1}, \cdot)} \right) \end{aligned}$$

Appendix C

It is straightforward to define the competitive equilibrium as well as the social optimum since the unique difference between the two models consists in the human capital accumulation process.

The Competitive Equilibrium: It is now described by the set $\{c_t^*, N_t^*, e_t^*, h_t^*, H_t^*, Y_t^*, w_t^*\}_{t=0}^{t=+\infty}$ satisfying equations $\{(38), (4) - (7), (15) - (17)\}_{t=0}^{t=+\infty}$.⁴⁴

The Social Optimum: It is now defined as the set $\{\widehat{c}_t, \widehat{N}_t, \widehat{e}_t, \widehat{h}_t, \widehat{H}_t, \widehat{Y}_t\}_{t=0}^{t=+\infty}$ satisfying equations $\{(38), (5), (6), (22), (24), (25)\}_{t=0}^{t=+\infty}$ and the following first-order condition with respect to $h_{t+1} \forall t$:

$$A(1 - [\phi N_{t+1} + \theta e_{t+1}] N_{t+1}) \left(u'_{c_{t+1}} + \mu_i \frac{q_{t+1}(1 - q_{t+1}) N_{t+1}}{2} u'''_{nn c_{t+1}} \right) + \eta_{t+1} [f'_2(e_{t+1}, \cdot, \cdot) + f'_3(e_{t+1}, \cdot, \cdot)] = \frac{\eta_t}{\beta} \quad (.55)$$

Then, I introduce the same economic policy as in the benchmark model: $\{\lambda_t, \Lambda_t, T_t\}_{t=0}^{t=+\infty}$ and I use the same method as in section 5. I obtain:

$$\widehat{\lambda}_t = \mathcal{A}_t \widehat{\lambda}_{t+1} + \mathcal{D}_t \epsilon_3^{f(e_{t+1}, h_t, \bar{h}_t)} + \mathcal{G}_t (\mu_i - 1) \quad (.56)$$

$$\widehat{\Lambda}_t = -\mathcal{J}_t \widehat{\lambda}_{t+1} - \mathcal{L}_t \epsilon_3^{f(e_{t+1}, h_t, \bar{h}_t)} + \mathcal{R}_t (\mu_i - 1) \quad (.57)$$

44. Obviously, any $f(e_t, h_t)$ becomes $f(e_t, h_t, \bar{h}_t)$.

with

$$\begin{aligned}\mathcal{D}_t &\equiv \frac{\beta N_{t+1} h_{t+1} (u'_{c_{t+1}} + \mu_t \varpi_{t+1}) f'_1(e_t, \cdot)}{N_t h_t (u'_{c_t} + \varpi_t) f'_1(e_{t+1}, \cdot)} \\ \mathcal{L}_t &\equiv -\theta e_t \mathcal{D}_t\end{aligned}$$

Appendix D

From (41), (42) and Definition 3, it is straightforward that:

$$\hat{\lambda} > 0 \iff \epsilon_3^f > \mathcal{W} \quad (.58)$$

$$\hat{\Lambda} > 0 \iff \epsilon_3^f > \mathcal{Z} \quad (.59)$$

From proposition 1, I know that $(\mathcal{W}, \mathcal{Z}) < (0, 0)$ if parents overshoot their optimal number of children. Because $\epsilon_3^{f(e_{t+1}, h_t, \bar{h}_t)} > 0$, it is straightforward that $(\hat{\lambda}, \hat{\Lambda}) > (0, 0)$ when parents have a precautionary demand for children.

From proposition 1, I know that $(\mathcal{W}, \mathcal{Z}) > (0, 0)$ if parents undershoot their optimal number of children. After some straightforward calculus, it appears that $\mathcal{W} < \mathcal{Z}$ is satisfied if and only if:

$$\pi \equiv 1 + \frac{\beta}{N} - \epsilon_2^f > 0 \quad (.60)$$

This is always satisfied. Then, when parents undershoot their optimal number of children, $\mathcal{W} < \mathcal{Z}$.

From conditions (.58) and (.59), it is straightforward that it is optimal to:

- tax education and subsidize births when $\epsilon_3^f < \mathcal{W}$
- subsidize both education and births when $\mathcal{Z} > \epsilon_3^f > \mathcal{W}$
- subsidize education and tax births when $\epsilon_3^f > \mathcal{Z}$

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- P. VAN HENTENRYCKE and L. WOLSEY (eds.) (2007), *Integration of AI and OR techniques in constraint programming for combinatorial optimization problems*. Berlin, Springer.
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- J-M. HURIOT and J-F. THISSE (eds) (2009), *Economics of cities*. Cambridge, Cambridge University Press.
- P. BELLEFLAMME and M. PEITZ (eds) (2010), *Industrial organization: markets and strategies*. Cambridge University Press.
- M. JUNGER, Th. LIEBLING, D. NADDEF, G. NEMHAUSER, W. PULLEYBLANK, G. REINELT, G. RINALDI and L. WOLSEY (eds) (2010), *50 years of integer programming, 1958-2008: from the early years to the state-of-the-art*. Berlin Springer.

CORE Lecture Series

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